

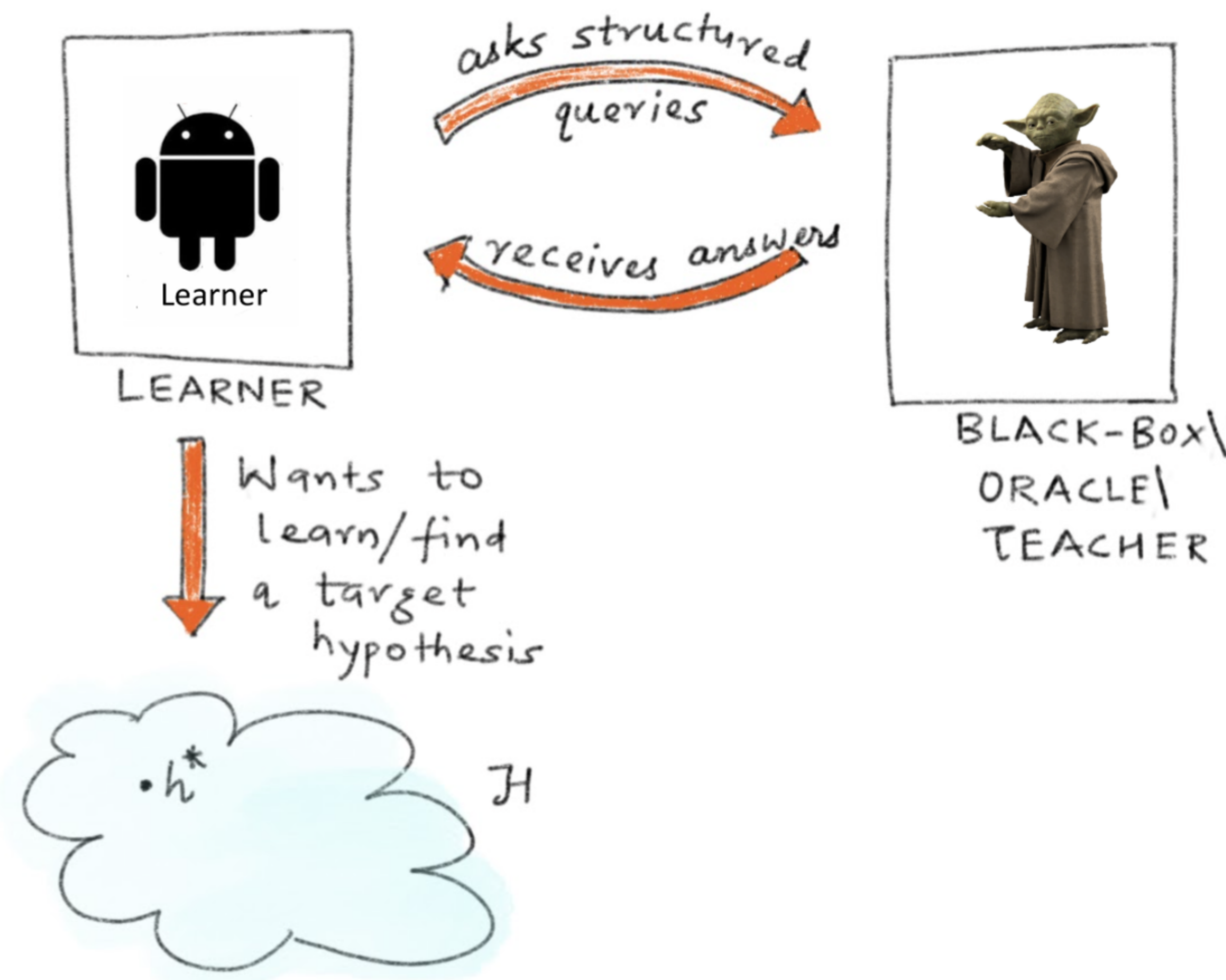
TEACHING VIA BEST-CASE COUNTEREXAMPLES IN THE LEARNING-WITH-EQUIVALENCE-QUERIES PARADIGM

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Learning with Queries Paradigm

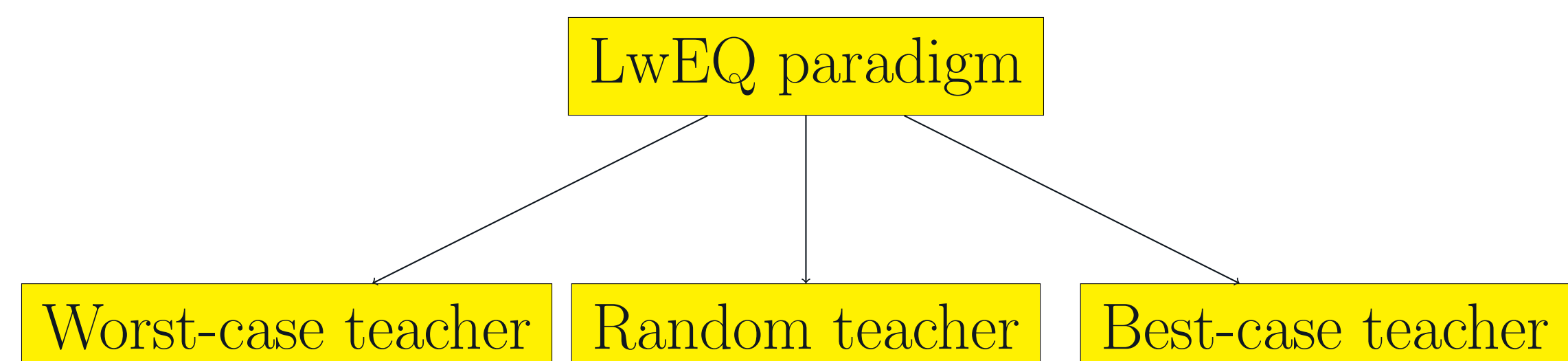
Learning with queries paradigm involves a learner asking structured queries to locate a target within a hypothesis class, and receiving answers from an oracle/teacher.



Teaching in the LwEQ Paradigm

Equivalence Query Protocol

- 1: Learner's initial hypothesis is $h_0 \in \mathcal{H}$, history is $Z_0 = \emptyset$, and version space is $\mathcal{H}_0 = \mathcal{H}$;
- 2: **for** $t = 1, 2, 3, \dots$ **do**
- 3: learner picks $h_t \in \mathcal{H}(Z_{t-1})$ based on Z_{t-1} and h_{t-1} ;
- 4: learner performs an equivalence query with h_t ;
- 5: teacher provides a response r_t that is either a "yes" or a "no" along with a counterexample z_t ;
- 6: **if** r_t is "yes" **then** learner identifies h^* and stops;
- 7: **else** learner updates $Z_t = Z_{t-1} \cup \{z_t\}$;



Comparisons and Connections

Table 1: Prior work on different settings in the context of LwEQ and LfS paradigms.

	Teaching	Worst-case Teacher	Random-case Teacher
Learning			
Learning-with-equivalence-queries (LwEQ)		Worst-case counterexamples [Ang87; AHK89; TM89]	Random counterexamples [AD17; Bha21]
Learning-from-samples (LfS)		Worst-case examples (i.e., least informative)	i.i.d learning [Val84; Vap99]

Table 2: Best-case teaching in the context of LwEQ and LfS paradigms.

	Teaching	Best-case Teacher
Learning		
Learning-with-equivalence-queries (LwEQ)		LwEQ-TD This work
Learning-from-samples (LfS)		LfS-TD / classical TD [GK95; Zil+11; Gao+17; KSZ19; Man+19]

LwEQ Paradigm and Query Complexity

- ▶ **Learning-with-equivalence-queries** (LwEQ) paradigm: learner is only asking equivalence queries and the teacher responds to a query either "yes" or "no" along with a counterexample on which the current hypothesis disagrees with the target hypothesis.
- ▶ **Query complexity** for the LwEQ paradigm: the number of equivalence queries needed by the learner for different types of teachers (varying in terms of informativeness of the counterexamples).

Our Contributions

A novel framework capturing the teaching process via preference functions Σ , where each function $\sigma \in \Sigma$ induces a teacher-learner pair. Our main results are as follows:

1. Characterize the optimal query complexity (teaching dimension) in the LwEQ paradigm, termed as LwEQ-TD.
2. Showcase the power of the best-case teacher in comparison to the worst-case and random teacher for the LwEQ paradigm.
3. Establish connections between LwEQ-TD with existing notions of TD for the learning-from-samples paradigm.

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