Teaching via Best-Case Counterexamples in the Learning-with-Equivalence-Queries Paradigm

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\textbf{Teaching in the LwEQ Paradigm}

\textbf{Equivalence Query Protocol}

1. Learner’s initial hypothesis is \(h_0 \in \mathcal{H}\), history is \(Z_0 = \varnothing\), and version space is \(\mathcal{H}_0 = \mathcal{H}\);
2. for \(t = 1, 2, 3, \ldots\) do
3. learner picks \(h_t \in \mathcal{H}(Z_{t-1})\) based on \(Z_{t-1}\) and \(h_{t-1}\);
4. learner performs an equivalence query with \(h_t\);
5. teacher provides a response \(r_t\) that is either a “yes” or a “no” along with a counterexample \(z_t\);
6. if \(r_t\) is “yes” then learner identifies \(\mathcal{H}\) and stops;
7. else learner updates \(Z_t = Z_{t-1} \cup \{z_t\}\);
8. end

\textbf{LwEQ paradigm:}

\begin{itemize}
  \item \textbf{Worst-case teacher}
  \item \textbf{Random teacher}
  \item \textbf{Best-case teacher}
\end{itemize}

\textbf{Our Contributions}

A novel framework capturing the teaching process via preference functions \(\Sigma\), where each function \(\sigma \in \Sigma\) induces a teacher-learner pair. Our main results are as follows:

1. Characterize the optimal query complexity (teaching dimension) in the LwEQ paradigm, termed as LwEQ-TD.
2. Showcase the power of the best-case teacher in comparison to the worst-case and random teacher for the LwEQ paradigm.
3. Establish connections between LwEQ-TD with existing notions of TD for the learning-from-samples paradigm.

\textbf{Comparisons and Connections}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Learning with equivalence queries (LwEQ)} & \textbf{Worst-case counterexamples} & \textbf{Random counterexamples} \\
\hline \textbf{Learning-from-samples (LFS)} & \textbf{Worst-case examples} & \textbf{11 of learning} \\
\hline \textbf{Learning read-once formulas with counter-examples} & \textbf{Learning from samples} & \textbf{Preference-based teaching} \\
\hline \textbf{Teaching} & \textbf{LwEQ-TD} & \textbf{This work} \\
\hline \textbf{Teaching} & \textbf{LwEQ-TD} & \textbf{This work} \\
\hline \textbf{Learning} & \textbf{LFS-TD / classical TD} & \textbf{Random} \textbf{case}\textbf{teacher} \\
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\end{tabular}
\end{table}

\textbf{References}